A Budget of Paradoxes. *By* PROFESSOR DE MORGAN.

(Continued from p. 108.)

No. XIV. 1830—1833.

1830. The celebrated interminable fraction 3·14159..., which the mathematician calls π, is the ratio of the circumference to the diameter. But it is thousands of things besides. It is constantly turning up in mathematics: and if arithmetic and algebra had been studied without geometry, π must have come in somehow, though at what stage or under what name must have depended upon the casualties of algebraical invention. As it is, our trigonometry being founded on the circle, π first appears as the ratio stated. If, for instance, a deep study of probable fluctuation from the average had preceded geometry, π might have emerged as a number perfectly indispensable in such problems as—What is the chance of the number of aces lying between a million $+x$ and a million $-x$, when six million of throws are made with a die? I have not gone into any detail of all those cases in which the paradoxer finds out, by his unassisted acumen, that results of mathematical investigation *cannot be:* in fact, this discovery is only an accompaniment, though a necessary one, of his paradoxical statement of that which *must be.* Logicians are beginning to see that the notion of *horse* is inseparably connected with that of *non-horse:* that the first without the second would be no notion at all. And it is clear that the positive affirmation of that which contradicts mathematical demonstration cannot but be accompanied by a declaration, mostly overtly made, that demonstration is false. If the mathematicians were interested in punishing this indiscretion, he could make his denier ridiculous by inventing asserted results which would completely take him in.

More than thirty years ago I had a friend, now long gone, who was a mathematician, but not of the higher branches: he was, *inter alia,* thoroughly up in all that relates to mortality, life assurance, &c. One day, explaining to him how it should be ascertained what the chance is of the survivors of a large number of persons now alive lying between given limits of number at the end of a certain time, I came, of course, upon the introduction of π, which I could only describe as the ratio of the circumference of a circle to its diameter. "Oh, my dear friend! that must be a delusion; what can the circle have to do with the numbers alive at the end of a given time?"—"I cannot demonstrate it to you;

but it is demonstrated."—"Oh! stuff! I think you can prove anything with your differential calculus: figment, depend upon it." I said no more; but, a few days afterwards, I went to him and very gravely told him that I had discovered the law of human mortality in the Carlisle table, of which he thought very highly. I told him that the law was involved in this circumstance. Take the table of expectation of life, choose any age, take its expectation and make the nearest integer a new age, do the same with that, and so on; begin at what age you like, you are sure to end at the place where the age past is equal, or most nearly equal, to the expectation to come. "You don't mean that this always happens?" —"Try it." He did try, again and again; and found it as I said. "This is, indeed, a curious thing; this *is* a discovery." I might have sent him about trumpeting the law of life: but I contented myself with informing him that the same thing would happen with any table whatsoever in which the first column goes up and the second goes down; and that if a proficient in the higher mathematics chose to palm a figment upon him, he could do without the circle: *à corsaire, corsaire et demi,* the French proverb says.

The first book of Euclid's Elements. With alterations and familiar notes. Being an attempt to get rid of axioms altogether; and to establish the theory of parallel lines, without the introduction of any principle not common to other parts of the elements. By a member of the University of Cambridge. Third edition. In usum serenissimæ filiolæ. London, 1830.

The author was Lieut.-Col. (now General) Perronet Thompson, the author of the *Catechism on the Corn Laws.* I reviewed the fourth edition—which had the name of *Geometry without Axioms,* 1833—in the quarterly *Journal of Education* for January, 1834. Colonel Thompson, who then was a contributor to—if not editor of—the *Westminster Review,* replied in an article the authorship of which could not be mistaken.

Some more attempts upon the problem, by the same author, will be found in the sequel. They are all of acute and legitimate speculation; but they do not conquer the difficulty in the manner demanded by the conditions of the problem.

Morning Post, Wednesday, May 4, 1831.

"We understand that although, owing to circumstances with which the public are not concerned, Mr. Goulburn declined becoming a candidate for University honours, that his scientific attainments are far from inconsiderable. He is well known to be the author of an essay in the Philosophical Transactions on the accurate rectification of a circular arc, and of an investigation of the equation of a lunar caustic—a problem likely to become of great use in nautical astronomy."

This hoax—which would probably have succeeded with any journal—was palmed upon the *Morning Post,* which supported Mr. Goulburn, by some Cambridge wags who supported Mr. Lubbock, the other candidate for the University of Cambridge. Putting on the usual concealment, I may say that I always suspected Dr-nkw-t-r B-th-n-e of having a share in the matter. The skill of the hoax lies in avoiding the words "quadrature of the circle," which all know, and speaking of "the accurate rectification of a circular arc," which all do not know, for its synonyme. The *Morning Post* next day gave a reproof to hoaxers in general, without referring to any particular case. It must be added that although there are *caustics* in mathematics, there is no *lunar* caustic.

So far as Mr. Goulburn was concerned, the above was poetic justice. He was the minister who, in the old time, told a deputation from the Astronomical Society that the Government "did not care twopence for all the science in the country." There may be some still alive who remember this: I heard it from more than one of those who were present, and are now gone. Matters are much changed. I was thirty years in office at the Astronomical Society; and, to my certain knowledge, every Government of that period, Whig and Tory, showed itself ready to help with influence when wanted, and with money whenever there was an answer for the House of Commons.

(*To be continued.*)

THINGS WORTH NOTING.

3.—SOMETHING MORE ABOUT DE MOIVRE'S FORMULA.

Mr. Baron Maseres, in the preface to his cumbrous work on the *Principles of the Doctrine of Life Annuities* (1783, quarto), at p. xi., after referring to and commending for its utility the method for finding the value of an annuity on (x) in terms of that of an annuity on $(x+1)$, has the following:—

"This method was first communicated to me by Dr. Price, but it was published in the year 1779, by Mr. William Morgan, the actuary to the Society for Equitable Assurances, near Blackfriars Bridge, in his *Treatise on the Doctrine of Annuities and Assurances on Lives*, pp. 56, 57; and it had been published before by Dr Price himself, in his *Treatise on Reversionary Payments*, Note O of the Appendix, and likewise by Mr. Thomas Simpson, in his book on *Life Annuities*, Prob. 1, Coroll. 7; which last book was published so long ago as the year 1742. But I should suspect that it was not known to Mr. De Moivre when he calculated his tables of the values